Mass-Energy-Momentum Radiation in Stueckelberg-Horwitz-Piron (SHP) Electrodynamics

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Background

Covariant canonical mechanics — Stueckelberg (1941), Horwitz and Piron (1970), Horwitz (1989)

8D unconstrained phase space $\left(x^\mu(\tau), \dot{x}^\mu(\tau)\right)$ $\qquad \dot{x}^\mu = dx^\mu/d\tau$

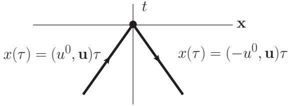
- Coordinate-independent evolution parameter τ: $[\tau, x^\mu] = 0$

Distinguish two aspects of time:

- Chronological time τ determines monotonic ordering of events
- Coordinate time x^0 locates event on laboratory clock

$$E = M\dot{x}^0 = M\frac{dx^0}{d\tau}$$

Particle: $\dot{x}^0 > 0$ $\qquad$ Antiparticle: $\dot{x}^0 < 0$



Upgrade nonrelativistic classical and quantum mechanics

$$\left.\begin{array}{c}\text{Newtonian time } t \\ + \\ \text{Galilean symmetry} \\ + \\ \text{Gauge} \quad p^\mu \rightarrow p^\mu + eA^\mu(x)\end{array}\right\} \longrightarrow \left\{\begin{array}{c}\text{Evolution parameter } \tau \\ + \\ \text{Poincaré symmetry} \\ + \\ \text{Gauge} \quad \begin{array}{c} p^\mu \rightarrow p^\mu + ea^\mu(x,\tau) \\ i\hbar\partial_\tau \rightarrow i\hbar\partial_\tau + ea^5(x,\tau)\end{array}\end{array}\right.$$

Dynamical U(1) gauge theory of spacetime events

- Event $x^\mu(\tau)$ evolves under chronological time $\tau \sim x^5$, $\qquad \mu,\nu = 0,1,2,3$
- $x^\mu(\tau) \longrightarrow$ a conserved 5D current $\partial_\alpha j^\alpha_\varphi(x,\tau) = 0$, $\qquad \alpha,\beta,\gamma = 0,1,2,3,5$
- $j^\alpha(x,\tau) \longrightarrow$ 10 field strengths $f^{\alpha\beta}(x,\tau)$ through pre-Maxwell equations

$$\partial_\beta f^{\alpha\beta}(x,\tau) = e j^\alpha_\varphi(x,\tau) \qquad \partial_\alpha f_{\beta\gamma} + \partial_\gamma f_{\alpha\beta} + \partial_\beta f_{\gamma\alpha} = 0$$

- Fields act on events through Lorentz force

$$M\ddot{x}^\mu(\tau) = e f^\mu{}_\alpha(x,\tau)\dot{x}^\alpha(\tau) = e\left[f^\mu{}_\nu(x,\tau)\dot{x}^\nu(\tau) + f^\mu{}_5(x,\tau)\right]$$

- Particles and fields can exchange mass through

$$\frac{d}{d\tau}\left(-\tfrac{1}{2}M\dot{x}^2\right) = -M\dot{x}^\mu\ddot{x}_\mu = e f_{5\mu}\dot{x}^\mu$$

Background

Mass-Energy-Momentum Tensor

Action

$$S = \int d\tau \; \frac{1}{2} M \dot{x}^\mu \dot{x}_\mu + \int d\tau d^4x \; \left\{ \frac{e}{c^2} \; j^\alpha(x,\tau) a_\alpha(x,\tau) - \int \frac{ds}{\lambda} \; \frac{1}{4c} \left[f^{\alpha\beta}(x,\tau) \Phi(\tau - s) f_{\alpha\beta}\left(x,s\right) \right] \right\}$$

Single particle current

$$j^\alpha(x,\tau) = c\dot{X}^\alpha(\tau) \delta^4\left(x - X(\tau)\right)$$

Current ensemble

$$\partial_\beta f^{\alpha\beta}(x,\tau) = \frac{e}{c} \int ds \; \varphi\left(\tau - s\right) \; j^\alpha(x,s) = \frac{e}{c} j^\alpha_\varphi(x,s)$$

$$\int \frac{d\tau'}{\lambda} \varphi\left(\tau - \tau'\right) \Phi(\tau' - s) = \delta\left(\tau - s\right) \qquad \varphi(\tau) = \lambda \int \frac{d\kappa}{2\pi} \, \frac{e^{-i\kappa\tau}}{1 + (\xi\lambda\kappa)^2} = \frac{1}{2\xi} e^{-|\tau|/\xi\lambda}$$

Noether current

$$\partial_\alpha T^{\alpha\beta}_\Phi = -\frac{e}{c^2} f^{\beta\alpha} j_\alpha \qquad T^{\alpha\beta}_\Phi = \frac{1}{\lambda c} \left[f^{\alpha\gamma}_\Phi f_\gamma{}^{\beta} + \frac{1}{4} f^{\delta\varepsilon}_\Phi f_{\delta\varepsilon} g^{\alpha\beta} \right]$$

Integrating with single particle current $\Longrightarrow$ total mass of particles and fields conserved

$$\frac{1}{c_5} \frac{d}{d\tau} \int d^4x \; T^{\alpha 5}_\Phi = \frac{e}{c} f^{5\mu}\left(X(\tau), \tau\right) \dot{X}_\mu(\tau) \quad \longrightarrow \quad \frac{d}{d\tau} \left[\frac{c_5}{c} \frac{1}{2} M \dot{x}^2 + g^{55} \int d^4x \; T^{55}_\Phi \right] = 0$$

Background

4-Vector Form

3D Maxwell equations

$$\nabla \cdot \mathbf{E} = \frac{e}{c} J^0 = e\rho \qquad \nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{B} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{E} = \frac{e}{c}\mathbf{J} \qquad \nabla \times \mathbf{E} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{B} = 0$$

4D pre-Maxwell equations

$$\partial_\mu f^{5\mu} = \frac{e}{c} j_\varphi^5 = \frac{ec_5}{c}\rho_\varphi \qquad \partial_\mu f_{\nu\rho} + \partial_\nu f_{\rho\mu} + \partial_\rho f_{\mu\nu} = 0$$

$$\partial_\nu f^{\mu\nu} - \frac{1}{c_5}\frac{\partial}{\partial \tau} f^{5\mu} = \frac{e}{c} j_\varphi^\mu \qquad \partial_\nu f_{5\mu} - \partial_\mu f_{5\nu} + \frac{1}{c_5}\frac{\partial}{\partial \tau} f_{\mu\nu} = 0$$

Wave equation and Green's function

$$\left(\partial^\mu \partial_\mu + g_{55}\frac{1}{c_5^2}\frac{\partial^2}{\partial \tau^2}\right) f^{\alpha\beta} = \partial^\alpha j^\beta - \partial^\beta j^\alpha$$

$$G_P(x,\tau) = -\frac{1}{2\pi}\delta(x^2)\delta(\tau) - \frac{c_5}{2\pi^2}\frac{\partial}{\partial x^2}\theta(-g_{55}g_{\alpha\beta}x^\alpha x^\beta)\frac{1}{\sqrt{-g_{55}g_{\alpha\beta}x^\alpha x^\beta}}$$

Background

Concatenation $\longrightarrow$ on-shell Maxwell theory $\sim$ equilibrium limit

Equilibrium boundary conditions

$$\rho_\varphi(x,\tau) \xrightarrow[\tau\to\pm\infty]{} 0 \qquad\qquad f^{5\mu}(x,\tau) \xrightarrow[\tau\to\pm\infty]{} 0$$

Integration over worldline

$$\left.\begin{array}{c} \partial_\nu f^{\mu\nu} - \dfrac{1}{c_5}\dfrac{\partial}{\partial\tau} f^{5\mu} = \dfrac{e}{c} j^\mu_\varphi \\[2ex] \partial_\mu f_{\nu\rho} + \partial_\nu f_{\rho\mu} + \partial_\rho f_{\mu\nu} = 0 \\[2ex] \partial_\alpha j^\alpha = 0 \end{array}\right\} \xrightarrow[\int \frac{d\tau}{\lambda}]{} \left\{\begin{array}{c} \partial_\nu F^{\mu\nu}(x) = \dfrac{e}{c} J^\mu(x) \\[2ex] \partial_\mu F_{\nu\rho} + \partial_\nu F_{\rho\mu} + \partial_\rho F_{\mu\nu} = 0 \\[2ex] \partial_\mu J^\mu(x) = 0 \end{array}\right.$$

where

$$A^\mu(x) = \int \frac{d\tau}{\lambda}\, a^\mu(x,\tau) \qquad F^{\mu\nu}(x) = \int \frac{d\tau}{\lambda}\, f^{\mu\nu}(x,\tau) \qquad J^\mu(x) = \int \frac{d\tau}{\lambda}\, j^\mu(x,\tau)$$

Green's function

$$\int d\tau\, G_P(x,\tau) = -\frac{1}{2\pi}\delta(x^2)$$

3-vector and scalar components

$$(\mathbf{e})^i = f^{0i} \qquad (\mathbf{b})_i = \epsilon_{ijk} f^{jk} \qquad (\boldsymbol{\epsilon})^i = f^{5i} \qquad \epsilon^0 = f^{50}$$

3D pre-Maxwell equations

$$\nabla \cdot \mathbf{e} - \frac{1}{c_5}\frac{\partial}{\partial \tau}\epsilon^0 = \frac{e}{c} j_\varphi^0 = e\rho_\varphi^0 \qquad \nabla \times \mathbf{e} + \frac{1}{c}\frac{\partial}{\partial t}\mathbf{b} = 0 \qquad \nabla \cdot \boldsymbol{\epsilon} + \frac{1}{c}\frac{\partial}{\partial t}\epsilon^0 = \frac{e}{c} j_\varphi^5 = \frac{ec_5}{c}\rho_\varphi$$

$$\nabla \times \mathbf{b} - \frac{1}{c}\frac{\partial}{\partial t}\mathbf{e} - \frac{1}{c_5}\frac{\partial}{\partial \tau}\boldsymbol{\epsilon} = \frac{e}{c}\mathbf{j}_\varphi \qquad \nabla \cdot \mathbf{b} = 0 \qquad \nabla \times \ \boldsymbol{\epsilon} - g^{55}\frac{1}{c_5}\partial_\tau \mathbf{b} = 0$$

$$\nabla \epsilon^0 + \frac{1}{c}\frac{\partial}{\partial t}\boldsymbol{\epsilon} + g^{55}\frac{1}{c_5}\frac{\partial}{\partial \tau}\mathbf{e} = 0$$

Mass-Energy-Momentum Tensor

$$T_\Phi^{55} = \frac{\lambda}{2c}\left[\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_\Phi - \epsilon^0 \epsilon_\Phi^0 + g^{55}\left(\mathbf{e} \cdot \mathbf{e}_\Phi - \mathbf{b} \cdot \mathbf{b}_\Phi\right)\right] \qquad T_\Phi^{5i} = \frac{\lambda}{c}\left(\boldsymbol{\epsilon} \times \mathbf{b}_\Phi + \epsilon^0 \mathbf{e}_\Phi\right)^i$$

$$T_\Phi^{00} = \frac{\lambda}{2c}\left[\mathbf{e} \cdot \mathbf{e}_\Phi + \mathbf{b} \cdot \mathbf{b}_\Phi + g_{55}\left(\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}_\Phi + \epsilon^0 \epsilon_\Phi^0\right)\right] \qquad T_\Phi^{0i} = \frac{\lambda}{c}\left(\mathbf{e} \times \mathbf{b}_\Phi + g_{55}\epsilon^0 \boldsymbol{\epsilon}_\Phi\right)^i$$

$$T_\Phi^{50} = \frac{\lambda}{c}\mathbf{e} \cdot \boldsymbol{\epsilon}_\Phi$$

Plane Waves
Sourceless Fields

Fourier Transform

$$f^{\alpha\beta}(x,\tau) = \frac{1}{(2\pi)^5}\int d^5k\; e^{ik\cdot x} f^{\alpha\beta}(k) = \frac{1}{(2\pi)^5}\int d^4k\; d\kappa\; e^{i(\mathbf{k}\cdot\mathbf{x}-k_0x^0+g_{55}c_5\kappa\tau)} f^{\alpha\beta}(k,\kappa)$$

$$\kappa = k^5 = g^{55}k_5$$

Sourceless pre-Maxwell equations

$$\mathbf{k}\cdot\mathbf{e} - g_{55}\kappa\epsilon^0 = 0 \qquad \mathbf{k}\cdot\mathbf{b} = 0 \qquad \mathbf{k}\cdot\boldsymbol{\epsilon} - k^0\epsilon^0 = 0$$

$$\mathbf{k}\times\mathbf{e} - k^0\mathbf{b} = 0 \qquad \mathbf{k}\times\mathbf{b} + k^0\mathbf{e} - g_{55}\kappa\boldsymbol{\epsilon} = 0$$

$$\mathbf{k}\times\boldsymbol{\epsilon} - \kappa\mathbf{b} = 0 \qquad -\kappa\mathbf{e} + k^0\boldsymbol{\epsilon} - \mathbf{k}\epsilon^0 = 0$$

Wave equation $\longrightarrow$ 5D mass-shell constraint

$$k^\alpha k_\alpha = \mathbf{k}^2 - (k^0)^2 + g_{55}\kappa^2 = 0$$

Concatenation $\longrightarrow$ $\kappa = 0$ $\longrightarrow$ 4D mass-shell constraint

$$c_5\int d\tau\; f^{\alpha\beta}(x,\tau) = \frac{1}{(2\pi)^4}\int d^4k\; e^{i(\mathbf{k}\cdot\mathbf{x}-k_0x^0)}\int d\kappa\;\delta(\kappa)\; f^{\alpha\beta}(k,\kappa)$$

Plane Waves

Solution

Independent 3-vector polarizations: $\boldsymbol{\epsilon}_\parallel$ and $\mathbf{e}_\perp$

$$\mathbf{e}_\parallel = g_{55}\frac{\kappa}{k^0}\boldsymbol{\epsilon}_\parallel \qquad \boldsymbol{\epsilon}_\perp = \frac{\kappa}{k^0}\mathbf{e}_\perp \qquad \epsilon^0 = \frac{1}{k^0}\mathbf{k}\cdot\boldsymbol{\epsilon}_\parallel \qquad \mathbf{b} = \frac{1}{k^0}\mathbf{k}\times\mathbf{e}_\perp$$

Components of $T^{\alpha\beta}$

Energy and mass density

$$T^{00} = \frac{\lambda}{c}\left[\mathbf{e}_\perp^2 + g_{55}\boldsymbol{\epsilon}_\parallel^2\right] \qquad T^{55} = \frac{\kappa^2}{k_0^2}T^{00}$$

Mass and energy flux — Poynting 3-vectors

$$T^{5i} \longrightarrow \mathbf{T}^5 = \frac{\kappa\mathbf{k}}{k_0^2}T^{00} \qquad T^{0i} \longrightarrow \mathbf{T}^0 = \frac{\mathbf{k}}{k^0}T^{00}$$

Mass — Poynting 4-vector

$$T^{5\mu} = \frac{\kappa k^\mu}{k_0^2}T^{00}$$

Uniformly Moving Charge

Maxwell Current

Rest frame

$$A(x) = \frac{e}{4\pi R} \qquad \mathbf{E}(x) = -\nabla A = \frac{e}{4\pi R^2}\hat{R} \qquad \mathbf{B}(x) = \nabla \times A = 0$$

$$T^{00} = \frac{1}{2}\left(\mathbf{E}^2 + \mathbf{B}^2\right) = \frac{1}{2}\left(\frac{e}{4\pi R^2}\right)^2 \qquad \mathbf{T} = \mathbf{E} \times \mathbf{B} = 0$$

Uniform Motion

Charge moving as $r = u\tau = (\gamma c\tau, \gamma\beta c\tau, 0, 0) = \gamma c\tau\hat{\mathbf{t}} + \gamma\beta c\tau\hat{\mathbf{x}}$

Observation point $x = (0, a, b, 0) = a\hat{\mathbf{x}} + b\hat{\mathbf{y}}$

Field strengths

$$\mathbf{E} = -e\gamma\frac{a\hat{\mathbf{x}} + b\hat{\mathbf{y}}}{4\pi\left(a^2\gamma^2 + b^2\right)^{3/2}} \qquad \mathbf{B} = -e\gamma\beta\frac{\hat{\mathbf{x}} \times \hat{\mathbf{y}}}{4\pi\left(a^2\gamma^2 + b^2\right)^{3/2}}$$

Poynting vector

$$\mathbf{E} \times \mathbf{B} = \frac{(e\gamma)^2\beta}{(4\pi)^2\left(a^2\gamma^2 + b^2\right)^3}\left(b\hat{\mathbf{x}} - a\hat{\mathbf{y}}\right)$$

Enclose path in cylinder with area element $d\mathbf{S} = r\left(\cos\phi\hat{\mathbf{y}} + \sin\phi\hat{\mathbf{z}}\right)d\phi\, da$

Total energy flux

$$\int(\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{S} = -r\int_0^{2\pi}\cos\phi\, d\phi\int_{-\infty}^{\infty}da\, \frac{a\,(e\gamma)^2\beta}{(4\pi)^2\left(a^2\gamma^2 + b^2\right)^3} = 0$$

Uniformly Moving Charge

pre-Maxwell Current

Potential in rest frame

Event $X(\tau) = (c\tau, \mathbf{0}) \qquad X^5(\tau) = c_5\tau$

Current $j_\varphi(x,\tau) = \left(c\delta^3(\mathbf{x})\,\varphi(t-\tau), \mathbf{0}\right) \qquad j^5(x,\tau) = \frac{c_5}{c} j^5(x,\tau)$

Potential $a^0(x,\tau) = \frac{e}{4\pi R}\varphi(\tau - \tau_R) \qquad \mathbf{a}(x,\tau) = 0 \qquad a^5(x,\tau) = \frac{c_5}{c} a^0(x,\tau)$

Retarded time $\tau_R = t - R/c$

Field strengths

$$\mathbf{e} = -\nabla a^0 = \frac{e}{4\pi R}\left(\frac{\varphi}{R} - \frac{\varphi'}{c}\right)\hat{\mathbf{R}} \qquad \boldsymbol{\epsilon} = -\nabla a^5 = \frac{c_5}{c}\mathbf{e}$$

$$\mathbf{b} = 0 \qquad \epsilon^0 = g_{55}\frac{1}{c_5}\frac{\partial}{\partial\tau}a^0 + \frac{1}{c}\frac{\partial}{\partial t}a^5 = -\frac{e}{4\pi R}\left(g_{55} - \frac{c_5^2}{c^2}\right)\frac{\varphi'}{c_5}$$

Handling singularities

$$\varphi(\tau) = 1/\left(2\xi\right) e^{-|\tau|/\xi\lambda} \quad \longrightarrow \quad \varphi'(0) = 0$$

$$\mathbf{e}\cdot\mathbf{e}_\Phi \propto \left[\varphi(\tau - \tau_r) - (R/c)\varphi'(\tau - \tau_r)\right]\left[\delta(\tau - \tau_r) - (R/c)\delta'(\tau - \tau_r)\right] = \varphi(0)\delta(0)$$

Uniformly Moving Charge

Components of $T^{\alpha\beta}$

Mass and energy density

$$\int dx^0 \; T^{55} = \lambda \left(\frac{c_5^2}{c^2} + g^{55} \right) \frac{1}{2} \left(\frac{e}{4\pi R^2} \right)^2$$

$$\int dx^0 \; T^{00} = \lambda \left(1 + \frac{c_5^2}{c^2} g^{55} \right) \frac{1}{2} \left(\frac{e}{4\pi R^2} \right)^2$$

Mass and energy flux density into space direction

$$\mathbf{T}^5 = -\frac{\lambda}{c} \left(g_{55} - \frac{c_5^2}{c^2} \right) \left(\frac{e}{4\pi R} \right)^2 \frac{\varphi'(0)}{R c_5} \hat{\mathbf{R}} = 0 \qquad \mathbf{T}^0 = g_{55} \frac{c_5}{c} \mathbf{T}^5 = 0$$

Mass flux density into time direction

$$\int dx^0 \; T^{50} = \lambda \frac{c_5}{c} \left(\frac{e}{4\pi R^2} \right)^2$$

Oscillating current density

$$\mathbf{J}\left(\mathbf{x},t\right)=\mathbf{J}\left(\mathbf{x}\right)e^{i\omega t}\qquad\qquad \nabla\cdot\mathbf{J}+i\omega\rho=0$$

Vector potential

$$\begin{aligned}\mathbf{A}\left(\mathbf{x},t\right)&=\frac{1}{4\pi}\int d^3x'dt'\,\frac{1}{\left|\mathbf{x}-\mathbf{x}'\right|}\delta\left(t-t'+\frac{\left|\mathbf{x}-\mathbf{x}'\right|}{c}\right)\mathbf{J}\left(\mathbf{x}'\right)e^{i\omega t'}\\&=e^{i\omega t}\,\frac{1}{4\pi}\int d^3x'\,\frac{e^{-ik\left|\mathbf{x}-\mathbf{x}'\right|}}{\left|\mathbf{x}-\mathbf{x}'\right|}\,\mathbf{J}\left(\mathbf{x}'\right)=\mathbf{A}\left(\mathbf{x}\right)e^{i\omega t}\qquad,\qquad k=\frac{\omega}{c}=\frac{2\pi}{\lambda}\end{aligned}$$

Far field approximation

$$\mathbf{x}=r\hat{\mathbf{r}}\longrightarrow R=\left|\mathbf{x}-\mathbf{x}'\right|=\left(r^2+\left(\mathbf{x}'\right)^2-2r\hat{\mathbf{r}}\cdot\mathbf{x}'\right)^{1/2}\simeq r-\hat{\mathbf{r}}\cdot\mathbf{x}'$$

Dipole approximation

$$e^{-ik\left|\mathbf{x}-\mathbf{x}'\right|}=e^{-ik\left(r-\hat{\mathbf{r}}\cdot\mathbf{x}'\right)}\simeq e^{-ikr}$$

$$\mathbf{A}\left(\mathbf{x}\right)=\frac{e^{-ikr}}{4\pi r}\int d^3x'\,\mathbf{J}\left(\mathbf{x}'\right)=-\frac{e^{-ikr}}{4\pi r}\int d^3x'\,\mathbf{x}'\,\nabla\cdot\mathbf{J}=\frac{e^{-ikr}}{4\pi r}\overbrace{i\omega\int d^3x'\,\mathbf{x}'\rho\left(\mathbf{x}'\right)}^{i\omega\mathbf{p}\;=\;Id\hat{\mathbf{d}}}$$

Simple Antenna in Maxwell Theory

Fields and Poynting vector

Magnetic field

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{Id}{4\pi}(-ik)\frac{e^{-ikr}}{r}\left(1 + \frac{1}{ikr}\right)\hat{\mathbf{r}} \times \hat{\mathbf{d}} \xrightarrow[1/kr \,\sim\, \lambda/r \,\to\, 0]{} \frac{\omega k}{4\pi}\frac{e^{-ikr}}{r}\hat{\mathbf{r}} \times \hat{\mathbf{p}}$$

Electric field

$$\mathbf{E} = \frac{1}{i\omega}\nabla \times \mathbf{B} = \frac{1}{4\pi}\left(k^2\frac{e^{-ikr}}{r}(\hat{\mathbf{r}} \times \mathbf{p}) \times \hat{\mathbf{r}} + [3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{p}) - \mathbf{p}]\left(\frac{1}{r^3} + \frac{ik}{r^2}e^{-ikr}\right)\right)$$

$$\mathbf{E} \xrightarrow[1/r^2 \,\ll\, 1/r]{} \frac{k^2}{4\pi}\frac{e^{-ikr}}{r}(\hat{\mathbf{r}} \times \mathbf{p}) \times \hat{\mathbf{r}} = \frac{k}{\omega}\mathbf{B} \times \hat{\mathbf{r}}$$

Take $\hat{\mathbf{d}} = \hat{\mathbf{z}} = \cos\theta\, \hat{\mathbf{n}} - \sin\theta\, \hat{\boldsymbol{\theta}}$

$$\mathbf{B} = \frac{\omega k}{4\pi}\frac{e^{-ikr}}{r}\sin\theta\, \hat{\boldsymbol{\phi}} \qquad \mathbf{E} = \frac{\omega k}{4\pi}\frac{e^{-ikr}}{r}\sin\theta\hat{\boldsymbol{\theta}} \qquad \mathbf{E} \times \mathbf{B}^* = \left(\frac{k}{4\pi}\right)^2\frac{(Id)^2}{r^2}\sin^2\theta\, \hat{\mathbf{r}}$$

Radiated power

$$P = \frac{1}{2}\int d\Omega\, r^2\, \hat{\mathbf{r}} \cdot \left[\left(\frac{k}{\omega}\mathbf{B} \times \hat{\mathbf{r}}\right) \times \mathbf{B}\right] = \frac{1}{2}\int_0^{2\pi} d\phi \int_0^{\pi}\left(\frac{\omega k}{4\pi}|\mathbf{p}|\right)^2 \sin^3\theta = \frac{k^2(Id)^2}{12\pi}$$

Simple Antenna in pre-Maxwell Theory

Pre-Maxwell current

Carry over approximation from Maxwell current $\mathbf{J}(\mathbf{x})$

$$j^0(x,\tau) = \left[J^0_{\text{background}}(\mathbf{x}) + J^0(\mathbf{x})\, e^{i\omega t}\right]\phi(\tau - t)$$

$$\mathbf{j}(x,\tau) = \mathbf{J}(\mathbf{x})\, e^{i\omega t}\phi(\tau - t)$$

$$j^5(x,\tau) = \frac{c_5}{c} j^0(x,\tau) = \frac{c_5}{c}\left[J^0_{\text{background}}(\mathbf{x}) + J^0(\mathbf{x})\, e^{i\omega t}\right]\phi(\tau - t)$$

Relationship of t and τ

$$\phi(\tau) = \frac{1}{2\sigma} e^{-|\tau|/\sigma} \qquad \phi(-\tau) = \phi(\tau) \qquad \int d\tau\, \phi(\tau) = 1 \qquad \phi'(\tau) = -\frac{\varepsilon(\tau)}{\sigma}\phi(\tau)$$

Background density ρ_0 insures positive charge and event densities

$$j^0(x,\tau) = \left[J^0_{\text{background}}(\mathbf{x}) + J^0(\mathbf{x})\, e^{i\omega t}\right]\phi(\tau - t) = c\left[\rho_0(\mathbf{x}) + \rho(\mathbf{x})\, e^{i\omega t}\right]\phi(\tau - t)$$

$$j^5(x,\tau) = \frac{c_5}{c} j^0(x,\tau) = c_5\left[\rho_0(\mathbf{x}) + \rho(\mathbf{x})\, e^{i\omega t}\right]\phi(\tau - t)$$

Simple Antenna in pre-Maxwell Theory

Applying Maxwell approximation to pre-Maxwell current

Maxwell current for collection of oscillating charges: $X_n(\tau) = \left(ct(\tau), \mathbf{a}_n e^{i\omega\tau}\right)$

$$\mathbf{J}(\mathbf{x}, t) = \sum_n \int d\tau\, \dot{\mathbf{X}}_n(\tau)\, \delta^4(x - X_n(\tau)) = \sum_n \int d\tau\, i\omega\, \mathbf{a}_n e^{i\omega\tau} \delta(ct - c\tau)\, \delta^3\left(\mathbf{x} - \mathbf{a}_n e^{i\omega\tau}\right)$$

$$= \left[\sum_n ik\, \mathbf{a}_n \delta^3\left(\mathbf{x} - \mathbf{a}_n e^{i\omega t}\right)\right] e^{i\omega t} \simeq \underbrace{\left[\frac{1}{T}\int_0^T dt\, \sum_n ik\, \mathbf{a}_n \delta^3\left(\mathbf{x} - \mathbf{a}_n e^{i\omega t}\right)\right]}_{\mathbf{J}(\mathbf{x})} e^{i\omega t}$$

Requirements for 5D pre-Maxwell current

$j^5(x,\tau)$ must be non-negative event density in spacetime

$$\partial_\mu j^\mu + \frac{1}{c_5}\frac{\partial}{\partial\tau} j^5 \quad \longrightarrow \quad \frac{1}{c_5}\frac{d}{d\tau}\int d^4x\, j^5 = \int d^4x\, \partial_\mu j^\mu = 0$$

$$j^5(x,\tau) = \dot{X}^5(\tau)\, \delta^4(x - X(\tau)) = c_5\, \delta^4(x - X(\tau)) \geq 0$$

$j^0(x,\tau)$ must be non-negative because $j^0(x,\tau) < 0 \ \Rightarrow\ \dot{t} < 0 \ \Rightarrow$ antiparticle

$$j^0(x,\tau) = c\dot{t}(\tau)\, \delta^4(x - X(\tau))$$

Simple Antenna in pre-Maxwell Theory

Current conservation

Conserved 5D current

$$\partial_\beta f^{\alpha\beta}(x,\tau) = \frac{e}{c}\, j^\alpha(x,\tau) \qquad \Rightarrow \qquad \partial_\alpha j^\alpha(x,\tau) = 0$$

Background density $\rho_0(\mathbf{x})$ independent of t and τ

$$\begin{aligned}
\frac{1}{c}\frac{\partial}{\partial t} j^0 + \nabla\cdot\mathbf{j} + \frac{1}{c_5}\frac{\partial}{\partial\tau} j^5 &= \rho\left(\mathbf{x}\right)\frac{\partial}{\partial t}\left[\left(e^{i\omega t}\right)\phi\left(\tau - t\right)\right] + \nabla\cdot\mathbf{J}\left(\mathbf{x}\right) e^{i\omega t}\phi\left(\tau - t\right) \\
&\quad + \rho\left(\mathbf{x}\right) e^{i\omega t}\frac{\partial}{\partial\tau}\phi\left(\tau - t\right) \\
&= i\omega\rho\left(\mathbf{x}\right) e^{i\omega t}\phi\left(\tau - t\right) - \underline{\rho\left(\mathbf{x}\right) e^{i\omega t}\phi'\left(\tau - t\right)} \\
&\quad + \nabla\cdot\mathbf{J}\left(\mathbf{x}\right) e^{i\omega t}\phi\left(\tau - t\right) + \underline{\rho\left(\mathbf{x}\right) e^{i\omega t}\phi'\left(\tau - t\right)} \\
&= \left[i\omega\rho\left(\mathbf{x}\right) + \nabla\cdot\mathbf{J}\left(\mathbf{x}\right)\right] e^{i\omega t}\phi\left(\tau - t\right)
\end{aligned}$$

$\rho\left(\mathbf{x}\right)$ and $\mathbf{J}\left(\mathbf{x}\right)$ related as in Maxwell theory

$$\rho\left(\mathbf{x}\right) = -\frac{1}{i\omega}\nabla\cdot\mathbf{J}\left(\mathbf{x}\right) \quad \longrightarrow \quad \frac{e}{c}\int d^3x\,\mathbf{J}\left(\mathbf{x}\right) = -\frac{e}{c}\int d^3x\,\mathbf{x}\,\nabla\cdot\mathbf{J} = \overbrace{\frac{i\omega}{c}\int d^3x\,\mathbf{x}e\rho\left(\mathbf{x}\right)}^{ik\mathbf{p}\;=\;Id\hat{\mathbf{d}}}$$

Simple Antenna in pre-Maxwell Theory

Total charge and current

Spacetime integral

$$Q(\tau) = \frac{e}{c^2}\int d^4x\; j^0(x,\tau) = \frac{e}{c^2}\int d^4x\; c\left[\rho_0(\mathbf{x}) + \rho(\mathbf{x})\, e^{i\omega t}\right]\phi(\tau - t)$$

$$= \int d^3x\; \rho_0(\mathbf{x}) \int dt\; \phi(\tau - t) + e\int d^3x\; \rho(\mathbf{x}) \int dt\; e^{i\omega t}\phi(\tau - t)$$

Using

$$\varphi(\tau - t) = \frac{1}{2\sigma} e^{-|\tau - t|/\sigma} = \int \frac{d\omega}{2\pi}\Phi(\omega)\, e^{i\omega(\tau - t)} \qquad\qquad \int d\tau\; \varphi(\tau - t) = 1$$

Total charge = background + oscillating charge

$$Q(\tau) = e\int d^3x\; \rho_0(\mathbf{x}) + e\int d^3x\; \rho(\mathbf{x})\;\; \Phi(\omega)\, e^{i\omega\tau} = Q_0 + Q\;\Phi(\omega)\, e^{i\omega\tau}$$

where

$$\Phi(\omega) = \frac{1}{1 + (\sigma\omega)^2} \qquad\qquad \rho(\mathbf{x}) = -\frac{1}{i\omega}\nabla\cdot\mathbf{J}(\mathbf{x})$$

Total current $\rightarrow$ oscillating dipole

$$e\mathbf{J}(\tau) = \int d^4x\; e\mathbf{J}(\mathbf{x})\, e^{i\omega t}\phi(\tau - t) = c\Phi(\omega)\, e^{i\omega\tau}\int d^3x\; e\mathbf{J}(\mathbf{x}) = i\omega c\;\Phi(\omega)\, e^{i\omega\tau}\;\mathbf{p}$$

Simple Antenna in pre-Maxwell Theory

Induced potential

Using dominant term $G_{Maxwell} = -\frac{1}{2\pi}\delta(x^2)\delta(\tau)$ in Green's function

$$a^\alpha(x,\tau) = \frac{e}{c}\int d^3x' \, \frac{1}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|} \, j^\alpha\left(c\left(t-\frac{\left|\mathbf{x}-\mathbf{x}'\right|}{c}\right),\mathbf{x}',\tau\right)$$

$$a^0(x,\tau) = e\int d^3x' \, \frac{\rho_0(\mathbf{x}')}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|}\phi\left(\tau - t + \frac{\left|\mathbf{x}-\mathbf{x}'\right|}{c}\right)$$
$$+e\int d^3x' \, \frac{\rho(\mathbf{x}')}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|}e^{i\omega\left(t-\frac{\left|\mathbf{x}-\mathbf{x}'\right|}{c}\right)}\phi\left(\tau - t + \frac{\left|\mathbf{x}-\mathbf{x}'\right|}{c}\right)$$

$$\mathbf{a}(x,\tau) = \frac{e}{c}\int d^3x' \, \frac{\mathbf{J}(\mathbf{x}')}{4\pi\left|\mathbf{x}-\mathbf{x}'\right|}e^{i\omega\left(t-\frac{\left|\mathbf{x}-\mathbf{x}'\right|}{c}\right)}\phi\left(\tau - t + \frac{\left|\mathbf{x}-\mathbf{x}'\right|}{c}\right)$$

$$a^5(x,\tau) = \frac{c_5}{c}a^0(x,\tau)$$

Simple Antenna in pre-Maxwell Theory

Far field and dipole approximations

In far field

$$\mathbf{x} = r\hat{\mathbf{r}} \qquad \longrightarrow \qquad \left|\mathbf{x} - \mathbf{x}'\right| = \sqrt{r^2 + (\mathbf{x}')^2 - 2r\hat{\mathbf{r}}\cdot\mathbf{x}'} \simeq r - \hat{\mathbf{r}}\cdot\mathbf{x}'$$

Dipole approximation — antenna small compared to oscillation wavelength

$$\left|k\hat{\mathbf{r}}\cdot\mathbf{x}'\right| < kd = \frac{2\pi d}{\lambda} \ll 1 \Rightarrow e^{ik\hat{\mathbf{r}}\cdot\mathbf{x}'} \simeq 1 \qquad \frac{r - \hat{\mathbf{r}}\cdot\mathbf{x}'}{c} \simeq \frac{r}{c}\left(1 - \hat{\mathbf{r}}\cdot\hat{\mathbf{x}}'\,\frac{d}{r}\right) \simeq \frac{r}{c}$$

Potentials

$$a^0(x,\tau) \simeq \frac{e}{4\pi r}\phi\left(\tau - t + \frac{r}{c}\right)\underbrace{\int d^3x'\ \rho_0(\mathbf{x}')}_{Q_0} + \frac{e}{4\pi r}e^{i\omega t}e^{-ikr}\phi\left(\tau - t + \frac{r}{c}\right)\underbrace{\int d^3x'\ \rho(\mathbf{x}')}_{Q}$$

$$= \frac{Q_0}{4\pi r} + \frac{Q}{4\pi r}\ e^{-i(kr-\omega t)}\ \phi\left(\tau - t + \frac{r}{c}\right)$$

$$\mathbf{a}(x,\tau) \simeq \frac{1}{4\pi r}e^{-ikr}e^{i\omega t}\frac{e}{c}\underbrace{\int d^3x'\ \mathbf{J}(\mathbf{x}')}_{\mathbf{p}}\phi\left(\tau - t + \frac{r}{c}\right)$$

$$= \mathbf{p}\ \frac{ik}{4\pi r}\ e^{-i(kr-\omega t)}\ \phi\left(\tau - t + \frac{r}{c}\right) \qquad\qquad a^5(x,\tau) = \frac{c_5}{c}a^0(x,\tau)$$

Simple Antenna in pre-Maxwell Theory

Field strengths

Using

$$\phi'(x) = -\frac{\varepsilon(x)}{\sigma}\,\phi(x) \qquad \frac{1}{kr} \ll 1 \qquad \mathbf{p} = \frac{1}{ik} Id\hat{\mathbf{d}}$$

Defining

$$\chi(x,\tau) = \frac{e^{-i(kr-\omega t)}}{4\pi r}\phi\left(\tau - t + \frac{r}{c}\right)$$

Field strengths

$$\mathbf{b} = \nabla \times \mathbf{a} = \widehat{\mathbf{b}}\,\chi \qquad \widehat{\mathbf{b}} = -ikId\left(1 + \frac{\varepsilon}{i\omega\sigma}\right)\,\hat{\mathbf{r}} \times \hat{\mathbf{d}}$$

$$\mathbf{e} = -\frac{1}{c}\frac{\partial}{\partial t}\mathbf{a} - \nabla a^0 = \frac{Q_0}{4\pi r^2}\,\hat{\mathbf{r}} + \widehat{\mathbf{e}}\,\chi \qquad \widehat{\mathbf{e}} = ik\left(1 + \frac{\varepsilon}{i\omega\sigma}\right)\left(Q\hat{\mathbf{r}} - Id\hat{\mathbf{d}}\right)$$

$$\boldsymbol{\epsilon} = g_{55}\frac{1}{c_5}\frac{\partial}{\partial \tau}\mathbf{a} - \frac{c_5}{c}\nabla a^0 = \frac{c_5}{c}\frac{Q_0}{4\pi r^2}\,\hat{\mathbf{r}} + \widehat{\boldsymbol{\epsilon}}\,\chi \qquad \widehat{\boldsymbol{\epsilon}} = ik\left[\frac{c_5}{c}\left(1 + \frac{\varepsilon}{i\omega\sigma}\right)Q\hat{\mathbf{r}} - g_{55}\frac{c}{c_5}\frac{\varepsilon}{i\omega\sigma}Id\hat{\mathbf{d}}\right]$$

$$\epsilon^0 = g_{55}\frac{1}{c_5}\frac{\partial}{\partial \tau}a^0 - \frac{1}{c}\frac{\partial}{\partial t}a^5 = \widehat{\epsilon}^{\,0}\chi \qquad \widehat{\epsilon}^{\,0} = ik\left[\frac{c_5}{c}\left(1 + \frac{\varepsilon}{i\omega\sigma}\right) - g_{55}\frac{c}{c_5}\frac{\varepsilon}{i\omega\sigma}\right]Q$$

Simple Antenna in pre-Maxwell Theory

Dimensionless parameters

Speeds of light

$$x^0 = ct \ , \ x^5 = c_5\tau \qquad \longrightarrow \qquad \frac{c_5}{c} \ll 1 = \text{ratio of rates of change in } \tau \text{ and } t$$

Time scales

$$e^{i\omega t} \ , \ \phi(\tau) = \frac{1}{2\sigma} e^{-|\tau|/\sigma} \qquad \longrightarrow \qquad \frac{1}{\omega\sigma} = \frac{T}{2\pi\sigma} = \frac{\text{antenna period}}{\text{correlation time}} \ll 1$$

Orientation of antenna

$$\hat{\mathbf{d}} = \hat{\mathbf{z}} \qquad \longrightarrow \qquad \hat{\mathbf{r}} \cdot \hat{\mathbf{d}} = \cos\theta$$

Polarizations

$$\widehat{\mathbf{e}} \simeq ik\left(Q\hat{\mathbf{r}} - Id\hat{\mathbf{z}}\right) \qquad \widehat{\mathbf{b}} \simeq -ikId\ \hat{\mathbf{r}} \times \hat{\mathbf{z}}$$

$$\widehat{\epsilon}^{\,0} \simeq ik\left[\frac{c_5}{c} - g_{55}\frac{c}{c_5}\frac{\varepsilon}{i\omega\sigma}\right] Q \qquad \widehat{\boldsymbol{\epsilon}} \simeq ik\left[\frac{c_5}{c} Q\hat{\mathbf{r}} - g_{55}\frac{c}{c_5}\frac{\varepsilon}{i\omega\sigma} Id\hat{\mathbf{z}}\right]$$

Simple Antenna in pre-Maxwell Theory

Bilinear terms in mass-energy-momentum tensor

Common spacetime dependence

$$\chi(x,\tau) = \frac{1}{4\pi r}\, e^{-i(kr-\omega t)}\, \phi\left(\tau - t + \frac{r}{c}\right)$$

Bilinear field combinations

$$T^{\alpha\beta} = \frac{1}{\lambda c}\left[f^{\alpha\gamma} f_{\gamma}^{\ \ \beta} + \frac{1}{4} f^{\delta\varepsilon} f_{\delta\varepsilon} g^{\alpha\beta}\right] \quad \longrightarrow \quad \mathrm{Re}\left[\left(A^{\alpha} + i\, B^{\alpha}\right)\chi\right] \times \mathrm{Re}\left[\left(C^{\beta} + i\, D^{\beta}\right)\chi\right]$$

Designate

$$S(x,\tau) = k^2 \left(\frac{\phi\left(\tau - t + \frac{r}{c}\right)}{4\pi r}\right)^2 \sin^2(kr - \omega t)$$

$$C(x,\tau) = k^2 \left(\frac{\phi\left(\tau - t + \frac{r}{c}\right)}{4\pi r}\right)^2 \cos^2(kr - \omega t)$$

$$X(x,\tau) = k^2 \left(\frac{\phi\left(\tau - t + \frac{r}{c}\right)}{4\pi r}\right)^2 2\sin(kr - \omega t)\cos(kr - \omega t)$$

$$S, C, X \sim \frac{1}{r^2}$$

Finite surface integrals at large r

Simple Antenna in pre-Maxwell Theory

Mass-energy-momentum tensor

$$T^{55} = \frac{\lambda}{2c}\left[g^{55}\left(Q - Id\ \cos\theta\right)^2 S\left(x,\tau\right) + \left(\frac{c}{c_5}\frac{\varepsilon}{\omega\sigma}\right)^2\left(\left(Id\right)^2 - Q^2\right) C\left(x,\tau\right) - g_{55}\frac{\varepsilon}{\omega\sigma} Q\left(Q - Id\ \cos\theta\right) X\left(x,\tau\right)\right]$$

$$T^{50} = \frac{\lambda}{c}\left[Q\frac{c_5}{c}\left(Q - Id\ \cos\theta\right)\ S\left(x,\tau\right) + g_{55}\frac{c}{c_5}\frac{\varepsilon}{\omega\sigma} Id\left(Id - Q\ \cos\theta\right)\ X\left(x,\tau\right)\right]$$

$$\mathbf{T}^{5} = \frac{\lambda}{c}\left[g_{55}\frac{c}{c_5}\frac{\varepsilon}{\omega\sigma} Id\left[Q - Id\ \cos\theta\right]\ X\left(x,\tau\right)\right]\ \hat{\mathbf{z}} + \frac{\lambda}{c}\left[g_{55}\frac{c}{c_5}\frac{\varepsilon}{\omega\sigma}\left(\left(Id\right)^2 - Q^2\right)\ X\left(x,\tau\right) + \frac{c_5}{c} Q\left(Q - Id\left(\cos\theta\right)\right)\ S\left(x,\tau\right)\right]\ \hat{\mathbf{r}}$$

$$T^{00} = \frac{\lambda}{2c}\left[\left(Q + Id\ \cos\theta\right)^2 + 2\left(Id\right)^2\left(1 - \left(\cos\theta\right)^2\right) + 2g_{55}\left(\frac{c_5}{c}\right)^2 Q^2\right] S\left(x,\tau\right) + \frac{\lambda}{2c}\left[\left(\frac{c}{c_5}\frac{\varepsilon}{\omega\sigma}\right)^2\left[\left(Id\right)^2 + Q^2\right] C\left(x,\tau\right) - \frac{\varepsilon}{\omega\sigma} Q\left[Q + Id\ \cos\theta\right] X\left(x,\tau\right)\right]$$

$$\mathbf{T}^{0} = \frac{\lambda}{c}\left[Id\left(Q - Id\ \cos\theta\right)\ S\left(x,\tau\right) - \frac{\varepsilon}{\omega\sigma} Q Id\left(X\left(x,\tau\right) + g_{55}\frac{c}{c_5} C\left(x,\tau\right)\right)\right]\ \hat{\mathbf{z}} + \frac{\lambda}{c}\left[\left[Id\left(Id - Q\ \cos\theta\right) + g_{55}\left(Q\frac{c_5}{c}\right)^2\right] S\left(x,\tau\right) - Q^2\frac{\varepsilon}{\omega\sigma}\ X\left(x,\tau\right)\right]\ \hat{\mathbf{r}}$$

Simple Antenna in pre-Maxwell Theory

Correlation of t and τ

Time structure

$$\phi\left(\tau-t\right)=\frac{1}{2\sigma}e^{-|\tau-t|/\sigma}$$

Imposes correlation	$\tau-t\sim\sigma$
Strong correlation	$\lim_{\sigma\rightarrow 0}\ \phi\left(\tau-t\right)=\delta\left(\tau-t\right)\quad\Rightarrow\quad t=\tau$
Weak correlation	$\sigma\ \rightarrow\ \text{large}\quad\Rightarrow\quad t-\tau\sim\text{evenly distributed}$

Potential under strong correlation

$$\begin{aligned}\mathbf{a}\left(x,\tau\right)&=\frac{e}{2\pi}\int d^3x'dt'\ \delta\left(\left(\mathbf{x}-\mathbf{x}'\right)^2-c^2\left(t-t'\right)^2\right)\ \mathbf{J}\left(\mathbf{x}'\right)e^{i\omega t'}\delta\left(\tau-t'\right)\\&=\frac{e}{4\pi c}\ e^{i\omega\tau}\int d^3x'\ \frac{1}{|\mathbf{x}-\mathbf{x}'|}\ \delta\left(\tau-t+\frac{|\mathbf{x}-\mathbf{x}'|}{c}\right)\ \mathbf{J}\left(\mathbf{x}'\right)\\&\simeq\frac{e}{4\pi rc}\ e^{i\omega\tau}\int d^3x'\ \delta\left(\tau-t+\frac{r-\hat{\mathbf{r}}\cdot\mathbf{x}'}{c}\right)\ \mathbf{J}\left(\mathbf{x}'\right)\end{aligned}$$

Coulomb-like potential on lightlike line from observation point to current

Instantaneous rigid τ-oscillation across spacetime on lightline

Suppression of waves controlled by $1/\omega\sigma$

Simple Antenna in pre-Maxwell Theory

Mass-energy-momentum tensor separating out terms in $1/\omega\sigma$

Mass density

$$T^{55} = \frac{\lambda}{2c} g^{55} \left(Q - Id \ \cos\theta \right)^2 S\left(x,\tau\right)$$

Mass transfer into spacetime

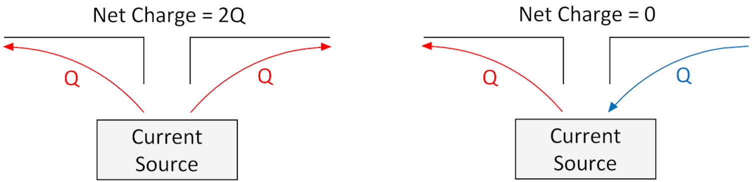
$$T^{50} = \frac{\lambda}{c} \frac{c_5}{c} Q \left(Q - Id \ \cos\theta \right) \ S\left(x,\tau\right) \qquad \mathbf{T}^5 = T^{50} \ \hat{\mathbf{r}}$$

Energy density

$$T^{00} = \frac{\lambda}{2c} \left[\left(Q + Id \ \cos\theta \right)^2 + 2 \left(Id \right)^2 \left(1 - \cos^2\theta \right) + 2 g_{55} \left(\frac{c_5}{c} \right)^2 Q^2 \right] S\left(x,\tau\right)$$

Energy transfer into space

$$\mathbf{T}^0 = \frac{\lambda}{c} \left[Id \left(Q - Id \ \cos\theta \right) \ \hat{\mathbf{z}} + \left(Id \left(Id - Q \ \cos\theta \right) + g_{55} \left(Q \frac{c_5}{c} \right)^2 \right) \ \hat{\mathbf{r}} \ \right] \ S\left(x,\tau\right)$$



Charge density

$$\rho(\mathbf{x}) = \begin{cases} \delta(x)\,\delta(y)\,\rho_z(z) & , \quad -\frac{d}{2} \le z \le \frac{d}{2} \\ 0 & , \quad \text{otherwise} \end{cases} \qquad \rho_z(-z) = -\rho_z(z)$$

Total charge

$$Q_{\text{total}} = \int d^3x\,\rho_0(\mathbf{x}) + \int d^3x\,\rho(\mathbf{x}) = Q_0 + \int_{-\frac{d}{2}}^{\frac{d}{2}} dz\,\rho_z(z) = Q_0$$

Total event number

$$N = \int d^4x\, j^5(x,\tau) = cc_5 \int d^3x dt\, \left[\rho_0(\mathbf{x}) + \rho(\mathbf{x})\, e^{i\omega t}\right] \phi(\tau - t) = cc_5 Q_0$$

Dipole moment

$$Id\,\hat{\mathbf{d}} = \frac{i\omega}{c} e \int d^3x\, \mathbf{x}\rho(\mathbf{x}) = 2ie\, k\hat{\mathbf{z}} \int_0^{\frac{d}{2}} dz\, z\,\rho_z(z)$$

Simple Antenna in pre-Maxwell Theory

Mass-energy-momentum tensor for neutral antenna

Mass density

$$T^{55} = \frac{\lambda}{2c} g^{55} \left(Q - Id \ \cos\theta\right)^2 S\left(x,\tau\right) \ \longrightarrow \ \frac{\lambda}{2c} g^{55} \left(Id\right)^2 \cos^2\theta \ S\left(x,\tau\right)$$

Mass transfer into spacetime

$$T^{50} = \frac{\lambda}{c} \frac{c_5}{c} Q \left(Q - Id \ \cos\theta\right) \ S\left(x,\tau\right) \ \longrightarrow \ 0 \qquad \mathbf{T}^5 = T^{50} \ \hat{\mathbf{r}} \ \longrightarrow \ 0$$

Energy density

$$T^{00} = \frac{\lambda}{2c} \left[\left(Q + Id \ \cos\theta\right)^2 + 2\left(Id\right)^2 \left(1 - \cos^2\theta\right) + 2g_{55} \left(\frac{c_5}{c}\right)^2 Q^2 \right] S\left(x,\tau\right)$$

$$\longrightarrow \ \frac{\lambda}{c} \left(Id\right)^2 \left(1 - \frac{1}{2}\cos^2\theta\right) S\left(x,\tau\right)$$

Energy transfer into space

$$\mathbf{T}^0 = \frac{\lambda}{c} \left[Id\left(Q - Id \ \cos\theta\right) \ \hat{\mathbf{z}} + \left(Id\left(Id - Q \ \cos\theta\right) + g_{55} \left(Q \frac{c_5}{c}\right)^2 \right) \ \hat{\mathbf{r}} \ \right] \ S\left(x,\tau\right)$$

$$\longrightarrow \ \frac{\lambda}{c} \left(Id\right)^2 \left(-\cos \ \hat{\mathbf{z}} + \hat{\mathbf{r}}\right) S\left(x,\tau\right) \ \Rightarrow \ \mathbf{T}^0 \cdot \hat{\mathbf{r}} = \frac{\lambda}{c} \left(Id\right)^2 \ \sin^2\theta \ S\left(x,\tau\right)$$

Simple Antenna in pre-Maxwell Theory

Mass conservation

Noether current

$$\partial_\alpha T^{\alpha 5} = -\frac{e}{c^2} f^{5\alpha} j_\alpha = -\frac{e}{c^2} \epsilon^\mu j_\mu$$

Taking $1/\omega\sigma \to 0$ and $Q = 0$

$$\widehat{\epsilon}^{\,0} = ik \left[\frac{c_5}{c} - g_{55} \frac{c}{c_5} \frac{\varepsilon}{i\omega\sigma} \right] Q \longrightarrow 0 \qquad \widehat{\boldsymbol{\epsilon}} = ik \left[\frac{c_5}{c} Q\hat{\mathbf{r}} - g_{55} \frac{c}{c_5} \frac{\varepsilon}{i\omega\sigma} Id\hat{\mathbf{z}} \right] \longrightarrow 0$$

$$T^{50} = \frac{\lambda}{c} \frac{c_5}{c} Q \left(Q - Id \cos\theta \right) S\left(x, \tau \right) \longrightarrow 0 \qquad \mathbf{T}^5 = T^{50}\, \hat{\mathbf{r}} \longrightarrow 0$$

Conservation of mass

$$\int d^4x\, \partial_\alpha T^{5\alpha} = \frac{1}{c_5} \frac{d}{d\tau} \int d^4x\; T^{55} + \frac{1}{c} \int d^4x\; \frac{\partial}{\partial t} T^{50} + \int d^4x\; \nabla \cdot \mathbf{T}^5 = -\frac{e}{c^2} \int d^4x\; \epsilon^\mu j_\mu$$

$$T^{5\alpha} = \mathbf{T}^5 = \epsilon^\mu = 0 \;\Rightarrow\; \frac{d}{d\tau} \int d^4x\; T^{55} = 0$$

Fields modified by time constraint

$$\hat{\epsilon}^{\,0} = 0 \qquad\qquad \hat{\boldsymbol{\epsilon}} \longrightarrow -g_{55} ik \frac{c}{c_5} \frac{\varepsilon}{i\omega\sigma} Id\hat{\mathbf{z}}$$

Additional terms in T^{55} and $T^{5\mu}$ field appear as mass density and transfer

Mass density
$$T^{55} = T_0^{55} + \frac{\lambda}{2c} \left(\frac{c}{c_5} \frac{\varepsilon}{\omega\sigma} \right)^2 (Id)^2 \, C\,(x,\tau)$$

Mass transfer into spacetime
$$T^{50} = -\frac{\lambda}{c} g_{55} \frac{c}{c_5} \frac{\varepsilon}{\omega\sigma} (Id)^2 \;\; X\,(x,\tau)$$

$$\mathbf{T}^5 = T^{50}\,(\hat{\mathbf{z}} - \hat{\mathbf{r}})$$

Energy density
$$T^{00} = T_0^{00} + \frac{\lambda}{2c} \left(\frac{c}{c_5} \frac{\varepsilon}{\omega\sigma} \right)^2 (Id)^2 \, C\,(x,\tau)$$

Energy transfer into spacetime
$$\mathbf{T}^0 = \mathbf{T}_0^0$$

Terms in $1/\omega\sigma$ conserved among themselves

Represent work required to produce $(\tau - t)$-correlated oscillation across spacetime